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ON THE REDUCED MHD FOR COMPRESSIBLE FLUIDS

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RESUMEN

Comúnmente, la turbulencia de la componente ionizada del Medio Interestelar es descrita en términos de la magnetohidrodinámica reducida (RMHD). Se muestra que dicha descripción no necesita que los gradientes en la dirección del campo magnético ambiente sean pequeños. Cuando el β del plasma es lejano a la unidad, la dinámica transversa governada por las ecuaciones de la RMHD puede coexistir, casi sin interacciones, con ondas de Alfvén no lineales que se propagan en la dirección del campo magnético ambiente. En cambio, para $\beta \approx 1$, los campos longitudinales (que en este caso no son despreciables) están acoplados con las ondas de Alfvén a pequeña escala, que no pueden ser filtradas.

ABSTRACT

Turbulence in the diffuse ionized component of the Interstellar Medium is often described in terms of the reduced magnetohydrodynamics (RMHD). We show that this description does not require small gradients in the directions of the ambient magnetic field. When the β of the plasma is far from unity, the transverse dynamics governed by the RMHD equations can coexist, with almost no interactions, with parallel-propagating nonlinear Alfvén waves. In contrast, for $\beta \approx 1$, the (no longer negligible) longitudinal fields are coupled with small-scale Alfvén waves that cannot be filtered out.

Key Words: **MHD — TURBULENCE — WAVES**

1. INTRODUCTION

Observations of radio wave scintillation in the Interstellar Medium (ISM) provide the evidence of anisotropic scattering and suggest a description in terms of anisotropic turbulence (see e.g., Frail et al. 1994; Spangler 1999). The relevance of this regime for the ionized phases of the ISM was first recognized by Higdon (1984) who proposed a model combining incompressible two-dimensional turbulence with non-propagating entropy variations, in order to explain observations of electron density fluctuations in the diffuse ISM (Armstrong, Cordes, & Rickett 1981).

Two-dimensional incompressible MHD turbulence in the planes transverse to the ambient field was proposed by Rosenbluth et al. (1976), Strauss (1976), and Montgomery (1982), as a reduced description of an incompressible MHD turbulence permeated by a strong uniform field. The approach was extended to weakly compressible MHD flows (Matthaeus & Brown 1988; Zank & Matthaeus 1992), and generalized to include spatial inhomogeneities (Bhattacharjee et al. 1998). In particular, these approaches get rid of nonlinear parallel-propagating Alfvén waves (AWs) that are usually simultaneously present in space plasmas and usually studied in the long-wavelength limit. In one space dimension, their dynamics is described by the Cohen-Kulsrud (1974) equations or, in the presence of dispersion due to the Hall effect, by the Derivative Nonlinear Schrödinger (DNLS) equation (Rogister 1971; Mjølhus 1976). This equation has been generalized to account for the presence of transverse variations (Mjølhus & Wyller 1988) and for the coupling with low-frequency magnetosonic waves parallel to the ambient field (Passot & Sulem 1993), an effect that plays a central role in the AW filamentation (Laveder, Passot, & Sulem 1999). It turns out that both phenomena can be captured in the same asymptotic framework (Gazol, Passot, & Sulem 1999; Champeaux et al. 2000), which enables us to revisit the validity conditions of RMHD.

2. COUPLING BETWEEN RMHD AND NONLINEAR ALFVÉN WAVES

The usual RMHD approach a priori assumes a regime where the variations in the longitudinal direction are much slower than in the transverse ones. For a compressible flow, this assumption allows one to eliminate the high-frequency waves and, depending on the squared ratio β of the sound and Alfvén speeds, to approximate the fluid motions by the 2D (or $2\frac{1}{2}$D when $\beta \approx 1$) incompressible MHD equations with linear longitudinal AWs of wavelengths larger than the characteristic scale of the transverse turbulence (Zank & Matthaeus 1992).

Instead, the scalings retained by Gazol et al. (1999) are chosen in a way that retains a nonlinear dynamics for the longitudinal AWs. Taking the Alfvén speed as unity, the regime of a strong ambient field is obtained by assuming small amplitude fluctuations. Using the squared Alfvénic Mach number $\epsilon = M_A^2$ as an expansion parameter, the transverse components of the velocity and magnetic fields are taken of order $\epsilon^{1/2}$. The sonic Mach number is thus given by $M_s = (\epsilon/\beta)^{1/2}$. The balance, between nonlinearity and Hall dispersion for the AWs dynamics is conveniently expressed in the reference frame moving at the Alfvén velocity, using a stretching of the spatial and temporal variables that depend on the parameter β. Far from the resonance between AWs and sound waves at $\beta = 1$, the longitudinal fields u_x and b_x and the density fluctuations $\rho - 1$ scale as ϵ, being thus of second order compared with the transverse components. In that case the longitudinal scale is stretched by a factor ϵ^{-1}. The ion Larmor radius being of order unity in the original variable, the Alfvén wavelength is then of order ϵ^{-1} compared with the ion Larmor radius. In contrast, when $\beta \approx 1$, all the fields are of the same order of magnitude and the Alfvén wavelength scales as $\epsilon^{-1/2}$ compared with the ion Larmor radius. The temporal scale τ over which nonlinearities start playing a role on the AW dynamics is proportional to ϵ^{-1} and ϵ^{-2} for $\beta \approx 1$ and $\beta \neq 1$ respectively. The two regimes are thus to be considered separately.

In both cases, denoting by ξ the stretched longitudinal variable and by η, ζ the transverse ones, the leading order of the MHD equations reads $\partial_\xi u + \partial_\xi b = 0$ where, up to a rescaling factor, the complex transverse fields are defined as $u = u_y + \mathrm{i}u_z$ and $b = b_y + \mathrm{i}b_z$. The coupling between the AWs and the turbulence is obtained by solving the above equations in the form $b = \tilde{b}(\xi,\eta,\zeta,\tau) + \bar{b}(\eta,\zeta,\tau)$ and $u = \tilde{u}(\xi,\eta,\zeta,\tau) + \bar{u}(\eta,\zeta,\tau)$, where the fluctuating parts (denoted by tildes) satisfy the usual AW condition $\tilde{u} = -\tilde{b}$, but where we also include mean contributions $\bar{u}$ and $\bar{b}$ resulting from averging over the ξ variable. It turns out that the transverse variations of both the fluctuating and mean fields take place on scales $\epsilon^{-3/2}$ when $\beta \neq 1$ (either small or large) or ϵ^{-1} when $\beta \approx 1$. Pushing the expansion to higher orders and writing the associated solvability conditions, one gets, to leading nontrivial order, the 2D incompressible MHD equations for the transverse mean fields, that are not affected by the other quantities. However, these mean fields affect the dynamics of the AWs and that of the mean longitudinal fields (low-frequency parallel magnetosonic waves) which in the case $\beta \neq 1$ are subdominant.

3. THE CASE β CLOSE TO UNITY

As already mentioned, for $\beta \approx 1$, the longitudinal fields are of the same order of magnitude as the transverse fields. The reductive perturbation expansion then leads to the mean field equations

$$\partial_T \bar{\mathbf{u}} + \bar{\mathbf{u}}\cdot\boldsymbol{\nabla}\bar{\mathbf{u}} = -\boldsymbol{\nabla} p + \bar{\mathbf{b}}\cdot\boldsymbol{\nabla}\bar{\mathbf{b}} \quad , \qquad \partial_T \bar{u}_x + \boldsymbol{\nabla}\cdot(\bar{u}_x\bar{\mathbf{u}} - \bar{b}_x\bar{\mathbf{b}}) = \frac{1}{2}\langle\langle \partial_\perp^*(\tilde{b}\tilde{\rho}) + \partial_\perp(\tilde{b}^*\tilde{\rho})\rangle\rangle \quad ,$$

$$\partial_T \bar{\mathbf{b}} - \boldsymbol{\nabla}\times(\bar{\mathbf{u}}\times\bar{\mathbf{b}}) = 0 \quad , \qquad \partial_T \bar{b}_x + \boldsymbol{\nabla}\cdot(\bar{b}_x\bar{\mathbf{u}} - \frac{1}{2}\bar{u}_x\bar{\mathbf{b}}) = 0 \quad ,$$

$$\boldsymbol{\nabla}\cdot\bar{\mathbf{u}} = 0 \quad , \qquad \boldsymbol{\nabla}\cdot\bar{\mathbf{b}} = 0 \quad , \qquad \bar{\rho} = -\bar{b}_x \quad ,$$

together with (R_i denotes the nondimensional gyromagnetic frequency of the ions appearing in the Hall term, $\beta - 1 = \epsilon^{1/2}\alpha$, $\partial_\perp = \partial_\eta + \mathrm{i}\partial_\zeta$ and $\boldsymbol{\nabla} = (\partial_\eta, \partial_\zeta)$, while the brackets $\langle\langle . \rangle\rangle$ hold for averaging over ξ and τ)

$$\partial_\tau \tilde{b} + \frac{1}{2}\partial_\xi(\tilde{\rho}\tilde{b}) - \frac{1}{2}\partial_\perp\tilde{\rho} + \frac{i}{2R_i}\partial_{\xi\xi}\tilde{b} + \partial_\xi[\tilde{b}(\bar{u}_x + \bar{b}_x - \frac{\bar{\rho}}{2})] = 0 \quad ,$$

$$2\partial_\tau\tilde{\rho} + \partial_\xi(\alpha\tilde{\rho} + \frac{1+\gamma}{2}\tilde{\rho}^2 + \frac{|\tilde{b}+\bar{b}|^2}{2}) - \frac{1}{2}(\partial_\perp^*\tilde{b} + \partial_\perp\tilde{b}^*) + [(\gamma-1)\bar{\rho} + 2\bar{u}_x]\partial_\xi\tilde{\rho} = 0 \quad .$$

The dominant pressure balance ($\bar{\rho} + \bar{b}_x = 0$) is associated with the incompressible character of the transverse flow when considered on the long time scale $T = \epsilon^{1/2}\tau$. Note that compressibility corrections affect the mean longitudinal magnetic field $\bar{b}_x$ even in the absence of small-scale AWs. This effect is at the origin of a coefficient discrepancy with the equation given in Zank & Matthaeus (1992). Furthermore, when small-scale Alfvén and magnetosonic waves in the longitudinal direction are retained, the large-scale dynamics is not decoupled, being affected by the mean effect of the waves, whose nonlinear evolution (on the short time scale τ) is itself sensitive to the mean fields. In the absence of mean fields, the equations given by Hada (1993) are recovered since the right hand side of the equation for $\bar{u}_x$ can be rewritten as a linear combination of $\bar{b}$ and $\bar{b}^*$.

4. CONCLUSIONS

For any value of β, in the presence of an intense uniform magnetic field, the dynamics of a compressible MHD flow decomposes, to leading order, into an incompressible 2D flow in the transverse directions and parallel-propagating nonlinear AWs that are affected by the transverse turbulence and drive longitudinal mean fields. The latter are subdominant when β is far from unity so that, up to large-scale linear AWs, RMHD is purely two-dimensional. We stress that this regime is based on the decoupling of the transverse flow from the longitudinal waves, allowed by the subdominant character of the longitudinal fields, and does not require an assumption of small longitudinal gradients (the parallel derivatives of the mean longitudinal fields are in fact comparable to the perpendicular derivative of the transverse components). In contrast, for $\beta \approx 1$, all the fields are of the same order and, in the presence of a mean transverse flow, the mean components of the longitudinal fields are coupled to nonlinear AWs that are usually present at small scale and cannot be filtered out.

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