NUMERICAL SIMULATION OF DYNAMOS WITH SCALE SEPARATION

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RESUMEN

El origen de los campos magnéticos en la astrofísica sigue siendo un reto para la teoría. La teoría del dinamo de campo medio se ha desarrollado desde hace ya algún tiempo pero tiene varios problemas. El crecimiento rápido del campo a escalas pequeñas, que supuestamente inhibe el aumento del campo a escalas grandes, es uno de los mayores problemas. Una salida a este dilema está en la teoría del dinamo rápido. Sin embargo esta teoría no ha podido desarrollarse tanto como uno quisiera, haciendo necesario el uso de simulaciones. Presentamos simulaciones con la resolución adecuada para separar las diferentes escalas. Se muestra que existen soluciones del dinamo donde el campo crece rápidamente a todas las escalas, incluyendo las mayores. Discutimos la energía que mantiene el espectro y la estructura de dichos dinamos.

ABSTRACT

Explaining the origin of magnetic fields in astrophysical bodies has long been a challenge for theorists. While mean field dynamo theory has been developed for a while this theory runs into several problems. The rapid growth of small scale magnetic fields which were thought to quench the emergence of larger scale fields was thought to be a major problem. A way out of this dilemma exists through invoking the theory of fast dynamos. However, the problem with fast dynamos is that the theory cannot be developed as much as one would like, making it necessary to rely on simulations. We provide simulations of dynamos with adequate resolution to have a separation of scales. Dynamo solutions are shown to be possible with magnetic fields growing rapidly on all scales including those larger than the forcing length scales. We discuss the energy bearing structures and spectra of such dynamos.

Key Words: **METHODS: NUMERICAL — MAGNETIC FIELDS — MHD — GALAXY: STRUCTURE**

1. INTRODUCTION

Classical dynamo theory was proposed as a way of explaining the growth of magnetic fields in astrophysical bodies (Parker 1955; Krause & Radler 1980), and has yielded many insights. Application of classical dynamo theory to bodies with large magnetic Reynolds number (Re_m), such as the Galaxy ($Re_m \sim 10^{19}$), has always been slightly problematical (Kulsrud & Anderson 1992). Using the LDIA approximation (Kraichnan 1965), Kulsrud & Anderson (1992) decomposed the magnetic field into a large scale and a small scale part. They found that the *small scale* magnetic field grows much faster than the large scale magnetic field. Thus the small scale magnetic fields are potentially capable of quenching the motions that could eventually have led to the formation of large scale magnetic fields.

Since large values of Re_m pose a problem, the idea of fast dynamos was proposed (Vainshtein & Zeldovich 1972; also see references in Childress & Gilbert 1995). Fast dynamos differ from classical dynamos in that the growth times are comparable to the characteristic eddy turnover times in the flow and thus might provide a way out of the dilemma. They have the problem, however, that their non-linear phase of evolution can only be traced by direct numerical simulations. There is the additional problem that fast dynamo theory is valid only in the limit of vanishingly small resistivity, thereby making direct simulation by a code that is non-linearly stable

very difficult. It is hoped that large simulations which have adequate scale separation between the forcing scales, the large scales (larger than the forcing scales), and the small scales (smaller than the forcing scales) might give us some insights on fast dynamo evolution. We present such simulations here.

The simulations were carried out on 256^3 zone periodic domains. The set-up is similar to that in Balsara & Pouquet (1999). An ABC forcing was applied to the velocity field on length scales of one sixth the computational domain. The forcing was such as to become decorrelated on a certain coherence time. The ratio of coherence time to eddy turn-over time ranged from 0.33 to 2.0, where the eddy turn-over time was 1.5 in code units. Over 30 eddy turnover times were simulated in each case. The forcing was such so as to maintain a Mach 1 flow through the course of the simulation, consistent with Mach numbers in the ISM. We have used the RIEMANN framework for computational astrophysics (see Balsara, these proceedings), for all the simulations. In § 2 we discuss energy evolution, in § 3 we discuss spectra and in § 4 we give a brief conclusion.

2. EVOLUTION OF ENERGY BEARING STRUCTURES

Figure 1a shows the evolution of the kinetic energy, denoted by (V), and the magnetic energy, (M), in a simulation where the ratio of coherence time to eddy turn-over time is 2.0. We see that the kinetic energy saturates rapidly. However, the magnetic energy shows an initial phase of exponential growth with a growth time of 3.06 code units. This time is comparable to the eddy turn-over time. Moreover, the simulations with very different coherence times also grew with the same time of growth. This is significant because according to the classical dynamo theory (see Krause & Radler 1980), the rate of growth should have been proportional to α^2 where the dynamo's α is proportional to the coherence time. Thus the different simulations, with vastly different coherence times, should have very different growth rates according to classical dynamo theory. Fast dynamo theory, on the other hand, predicts that their growth rate is comparable to the eddy turn-over time which is almost the same for all the simulations. Since the different simulations underwent a linear phase of growth with the same growth rate, it suggests that a fast dynamo is operating. When the magnetic energy reaches 2% of the kinetic energy, a slower quasi-linear phase of evolution sets in and its growth rate decreases. When the magnetic energy reaches 10% of the kinetic energy the evolution becomes strongly non-linear.
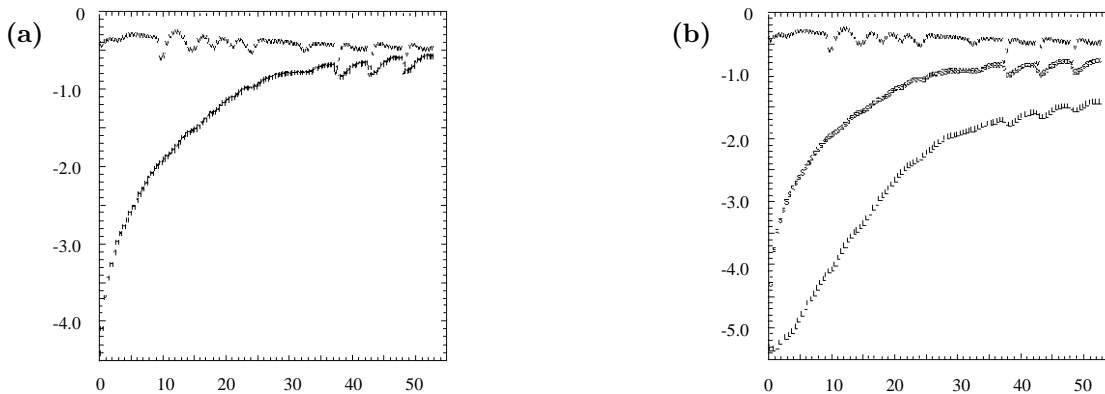

Fig. 1. *(a)* Time history of the log of kinetic (V) and magnetic (M) energies. *(b)* Same as *(a)* but with large scale (L) and small scale (S) magnetic energy shown separately.

In Figure 1b we plot the magnetic energy on scales larger than the forcing, denoted by L, and the magnetic energy on scales smaller than the forcing, (S), and also the kinetic energy, (V). We see that the magnetic energy on large and small scales grows at the same rate, but that on small scales saturates faster. The magnetic energy on larger scales approaches saturation much more slowly. However, the saturation of magnetic energy on the small scales does not prevent the magnetic energy on the large scales from growing. In view of the fact that the different simulations have almost the same linear rates of growth, it is interesting to ask how they differ. The answer is that the simulations with larger coherence times come much closer to equipartition between the magnetic and kinetic energies than simulations with smaller coherence times. Thus the eddy coherence time strongly influences the non-linear evolution of the field. In the non-linear phase of evolution it is significant to

notice that the turbulence undergoes several episodes of strong decline and replenishment. After each of those episodes, the small scale magnetic fields reach a strength that is comparable to their strength at the beginning of the episode. However, the large scale fields reach a strength that is a little greater than their strength at the beginning of the episode indicating that reorganization of large scale magnetic field structures is taking place.

3. EVOLUTION OF SPECTRA

Figure 2a shows several spectra of the magnetic energy plotted out on a log-log plot at equal time intervals. The legends "0" through "5" correspond to increasing time. The simulation has a ratio of coherence time to eddy turn-over time of 2.0 . The wave number of the forcing is given by $\log_{10}(k_{force}) = 1.15$. We see that the spectal energy on the small scales evolves rapidly, while the spectral energy on the larger scales grows very slowly. The largest modes in the simulation are prevented from growing because of effects associated with the finiteness of the computational domain. They are, therefore, excluded from spectral analysis.

Figure 2b shows the spectrum at the final time in the simulation. The small scales are best fit by the power law $E_M(k) = k^{-1.75}$, whereas the large scales are best fit by the power law $E_M(k) = k^{-2.7}$. Other runs in this series of simulations show similar spectral indices.
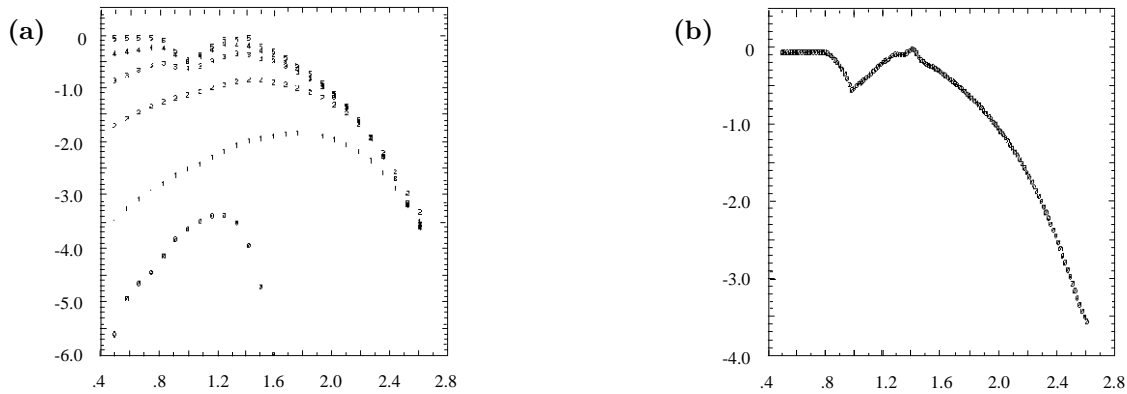

Fig. 2. *(a)* Several magnetic energy spectra at equal time intervals. *(b)* Final magnetic energy spectrum.

4. CONCLUSIONS

We have carried out several simulations of compressible dynamos. We notice that the time of growth is comparable to the eddy turn-over time during the linear phase. Both large and small scale magnetic fields grow at the same rate. The saturation of the small scale magnetic fields does not stop the large scale magnetic fields from growing. The coherence time of the eddies is seen to strongly influence the final saturated magnetic energies but not the growth rate during the linear phase of evolution. Spectral analysis provides further confirmation of this scenario. Spectral indices for small and large scale magnetic field structures are also derived.

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